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INTRODUCTION TO SIMULATOR

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INTRODUCTION TO SIMULATOR

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The field of application for simulators is extremely broad. Among the various applications, there are some which do not attract public attention but have a very high utilization rate in its specialized field. Furthermore, there are some which have been tried quite recently and which seem to be quite promising.

In this article, the writer would like to discuss flood simulators which have been discussed in relation with civil engineering from early times, simulators which analyse the response of high structure constructions against earthquakes, and piping network simulators which calculate the pressure and flow of huge piping networks like gas and water supply. After such descriptions, the writer would like to explain about the biological simulators which have attracted considerable attention quite recently, such as the human voice synthesizing appatuses and simulators for biological organs.

1. FLOOD SIMULATOR

It is a very important matter to highly utilize water which is a natural resource.

For instance, it has long been debated in the field of civil engineering as to how river flow should be adjusted and controlled for hydraulic power generator, irrigation, and prevention of flood.

Recently, this has been advanced a step further, and the river flow mechanism has been put into computers, and the detailed analysis of the river characteristics have been made together with programmed flow control. There is a tendency to control the river flow by a preprogrammed system, automatically, in accordance with the flow which changes momentarily.

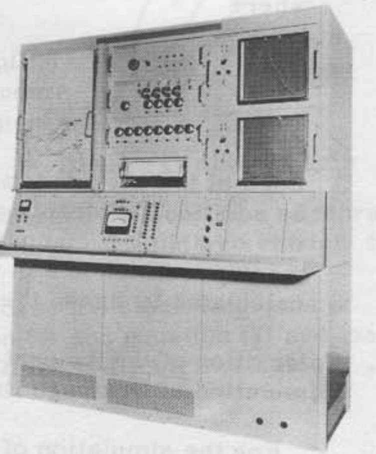

Fig. 1 Flood Simulator

Especially in recent years, the construction of multi-purpose dam and river repairing work have been advanced considerably, and the necessity of automatic and man-made control of river flow has demanded a clear grasping of the river characteristics. On the other hand, the rapid progress and development of the electronics have brought forth the development of analog computers and digital computer. The advent of these computers made possible the formulation of a "live mathematical model", and this made "simulators" possible.

Up to this time, "simulation" was applied chiefly to the control of man-made objects such as airplanes, missiles, and atomic reactors.

However, this has been advanced a step further, and how natural phenomena are being taken up as the object.

1.1. Various Methods of Flood Prediction Calculations

1.1.1. Calculation of the Amount of Rainfall

From the amount of rainfall at the drainage basin aimed at, and the region adjoining such a drainage basin, the typical rainfall of the drainage basin aimed at can be calculated from the following equations.

$$\left.\begin{aligned} R_1 &= f_1'R_A + f_1''R_B + f_1'''R_C \\ R_2 &= f_2'R_D + f_2''R_E + f_2'''R_f \\ &\cdot \\ &\cdot \\ &\cdot \\ R_m &= f_m'R_L + f_m''R_M + f_m'''R_N \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (1)$$

where

$R_1 \sim R_m$: Typical rainfall of each drainage basin.
$R_A \sim R_N$: Amount of rainfall in the vicinity of the drainage basin aimed at and which has influence on the drainage basin area aimed at.

$f_i{}' \sim f_i{}'''$: Typical rainfall coefficient (A coefficient for calculating the typical amount of rainfall of a certain drainage basin aimed at from the amount of rainfall in the drainage basin aimed at and the amount of rainfall in the adjoining drainage basin regions.)

In case of calculating the typical amount of rainfall in a certain drainage basin aimed at, the rainfall of the drainage basin aimed at is used.
Furthermore, the typical rainfall coefficient $f \sim f'''$ is handled as a constant specified by the statistical data of the past.

1.1.2. Calculation of the Outflow Coefficient

The outflow coefficient for obtaining the amount of flow from the rainfall function is generally given as a function of the total rainfall.
Calculate the total rainfall during the period in which the flood calculations are to be made, then obtain the outflow coefficient f(t) of that drainage basin region. Subsequently, use this f(t) and calculate the am ount of flow from the amount of rainfall.

$$f_i(t) = F_i\left(\sum_{i=1}^{64} R_i\right)$$

where

$f_i(t)$: Outflow coefficient of the drainage basin area i.
R_i : Amount of rainfall for the drainage basin area for each hour.
t : Elapsed time of rainfall.

Each of the abovementioned equations are composed of mathematical operations such as addition, subtraction, multiplication and division. Therefore, it is easier to handle this by a digital computer rather than an analog computer. As for the flood computing machines which are in practical use at present, although the small scale ones are calculated by hand, the large scale ones are calculated by the digital operation portion.

1.2. Calculation of the Amount of Flow (Simulation of Drainage Basin)

For the simulation of drainage basin which had rainfall, there are various methods. Each method has its own advantages and disadvantages. Therefore, at present, they are selected and utilized adequately in accordance with the specific characteristics of the river. In the following paragraphs, an explanation will be given for each method.

1.2.1. Storage Function Method

This is a method in which the calculation is made by the storage function based on the concept that the amount of outflow at a certain drainage basin or outflow end of a river can be expressed as a function of the storage amount of the drainage basin of the river.
The following function relationship exists between the inflow amount (1) at point B of the river shown in Fig. 2, and the outflow amount (O);

$$\frac{ds}{dt} = I - O \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3)$$

where

S : Storage amount (m^3)
I : Inflow amount (m^3/s)
O : Outflow amount (m^3/s)

From equation (3), the following equation can be drived.

$$\frac{ds}{dO} \cdot \frac{dO}{dt} = f \cdot I - O \qquad (4)$$

In case $\frac{ds}{dO} = \varphi(O)$ is substituted, equation (4) will become

$$\varphi(O)\frac{dO}{dt} = f \cdot I - O \qquad (5)$$

From equation (5)

$$\frac{dO}{dt} = \frac{1}{\varphi(O)}(f \cdot I - O) \qquad (6)$$

or

$$O = \int \frac{1}{\varphi(O)}(f \cdot I - O)dt \qquad (7)$$

where $\varphi(O)$ is called the storage function.
Generally speaking, part of the rainfall will not flow out on account of evaporation, interruption and ground moisture replenishment. In case rain falls on dry ground, it will wet the ground and part of the rainfall will be absorbed by the ground. Consequently, it will not flow to the river. Thus, it will become rainfall loss. Furthermore, part of it will penetrate into the ground, be stored, and gradually flow out and form the base amount (i. e. The amount which flows out regardless of whether it rains or not.) The amount of rainfall which exceeds the penetration rate will become surface storage (i. e. The amount of rain which is stored on the surface of the ground temporarily. This amount will flow into the river into the river in a very short period.), and eventually become the surface outflow.

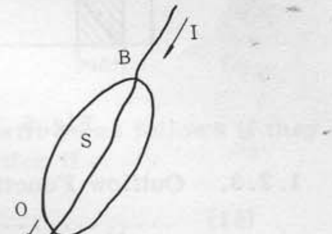

Fig. 2 Drainage basin

Besides the base outflow and surface outflow, there is also the intermediate outflow. This outflow can be expressed as a function of storage in the same way as the river. In case this relationship is illustrated by a model, it will be as shown in Fig. 3.

In case the block diagrams of equation (6) and equation (7) are made, they will be as illustrated below:

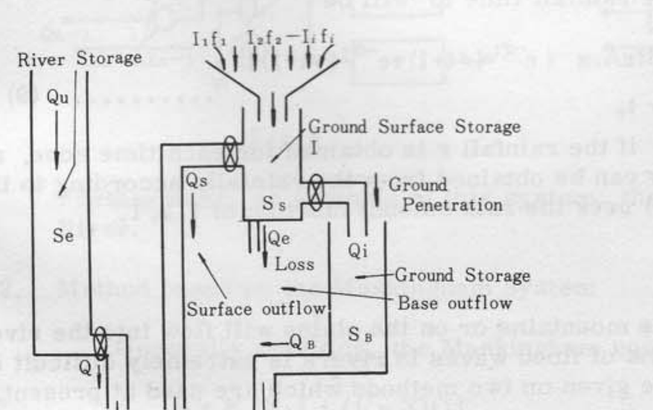

Fig. 3 Model Diagram of Inflow and Outflow

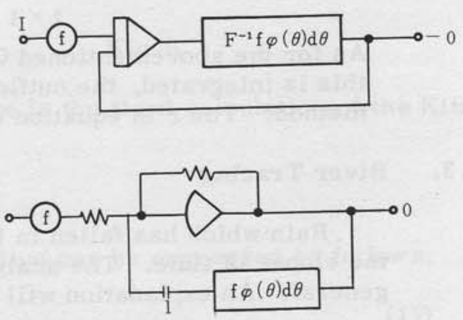

Fig. 4. Block Diagram based on the Storage Function Method.

In case of the above, if (O) = constant, the two circuits shown above will be substantially the same, and become the so called primary delay.

1.2.2. Unit Graph Method

In case there is reinfall of a certain unit in the upper stream of the drainage basin, the outflow from the lower stream is assumed by a certain pattern according to this method. If the rainfall continues, the outflow is calculated by shifting the unit time in accordance with the intensity of rain. In other words, the amount of outflow versus the rainfall is assumed to be linear.
However, in the case of an actual river, this assumption is not necessarily correct, and generally speaking, the relation between the two is non-linear. In spite of this, since the computing machine will be simplified, it is found to be quite useful as a means for obtaining a rough idea of the outflow. As a case in which the storage function method and unit graph method have been actually applied, the flood computing machine of The River Ishikari, Hokkaido can be given.
A unit graph which was utilized for this is shown in Fig. 6.

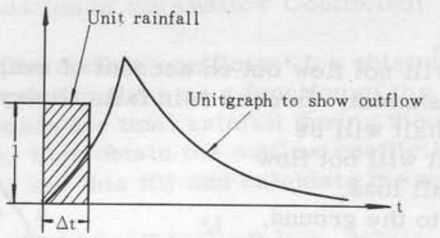

Fig. 5 Unit Graph

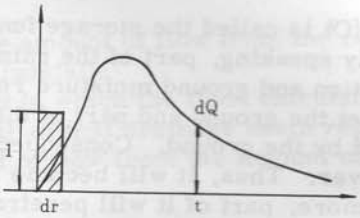

Fig. 6

1.2.3. Outflow Function Method

If we assume that a unit amount of rain falls on the drainage basin having a surface area of A in a short period of time d , the change dQ in outflow owing to this rain will be as follows.

$$dQ = Aate^{-\alpha t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

where a, α are constants.

If we represent the outflow coefficient by f, and take the units as A(km^2), r(mm/h), Q(m^3/sec), the outflow for the rainfall time to will be

$$Q = \int_0^{t_0} dQdt = 0.2773frA \times \{e^{-\alpha t_0}(\alpha t+1) - e^{-\alpha t}(\alpha t+1)\}$$
$$t = t - t_0 \quad \ldots\ldots\ldots \quad (9)$$

As for the abovementioned Q, if the rainfall r is obtained for each time zone, and this is integrated, the outflow can be obtained from the rainfall, according to this method. The r in equation (9) uses the rain amount function of 1.2.1.

1.3. River Tracing

Rain which has fallen in the mountains or on the plains will flow into the river with the elapse of time. The analysis of flood waves in rivers is extremely difficult in general. An explanation will be given on two methods which are used at present.

1.3.1. Method based on Equations of Motion and Continuity

The equation of motion for a non-constant flow in an open water channel can be expressed in general as follows.

$$\frac{\partial H}{\partial X} + \frac{V^2}{C^2R} + \frac{1}{g}\frac{\partial V}{\partial t} + \frac{\partial}{\partial X}\left(\frac{\alpha V^2}{2g}\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

However, since the flood waves are chiefly taken up as the object, item 3 and item 4 are neglected. Therefore,

$$\frac{\partial H}{\partial X} + \frac{V^2}{C^2 R} = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (11)$$

On the other hand, equation of continuity which puts into consideration the transverse inflow can be expressed as follows.

Q and H can be obtained by making simulateneous equations from the above two and obtaining the solution. However, by putting into consideration the solution by analog computers, the equations shall be converted into difference equations. The river is divided into n parts, and at cross section n, the equation using the middle step difference will be as shown below.

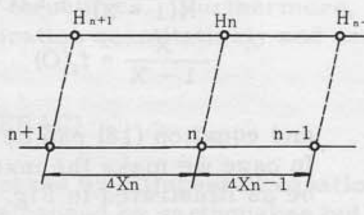

Fig. 7

$$\begin{cases} -Fn\left(\dfrac{H_{n+1} - H_{n-1}}{\Delta X_n + \Delta X_{n-1}}\right) = (Q_n)^2 & (13) \\[2ex] \dfrac{dAn}{dt} + \dfrac{Q_{n+1} - Q_{n-1}}{\Delta X_n + \Delta X_{n-1}} = q_n \cdots\cdots & (14) \end{cases}$$

where

$$F = \frac{1}{n^2} A^2 R^{\frac{4}{3}} \qquad A = f(H)$$

n : Coefficient of roughness

The boundary condition at the upper stream of (13) and (14) will be as follows if they are set from the amount of outflow Q' and its functional relation H.

$$Q_0 = Q' \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (15)$$

$$H_0 = \phi\,(Q') \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (16)$$

If a block diagram illustrating the above mentioned results are shown for one zone, it will be as shown in Fig. 8 for equation (14), and as Fig. 9 for equation (13).

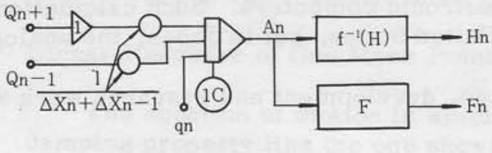

Fig. 8

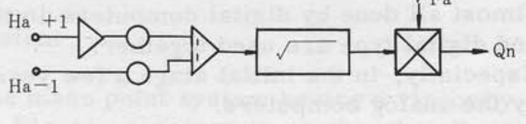

Fig. 9

Furthermore, an example of this system, there is the flood calculation of the Kitakami River.

1.3.2. Method based on the Maskingham System

Generally speaking, the Maskingham equation can be expressed as follows.

$$S = K\{X \cdot I + (1 - X)O\} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (17)$$

K and X are constants which are determined by experiments.

In order to make this into a simultaneous equation with the equation of continuity, the following conversion shall be made.

Equation (17) can be changed as follows.

$$O = \frac{1}{K(1 - X)} \cdot s - \frac{X}{1 - X} \cdot I \quad \ldots. (18)$$

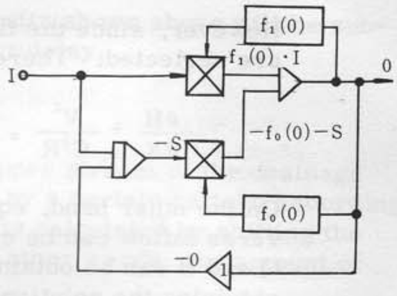

Fig. 10

On the other hand,

$$\frac{ds}{dt} = I - O \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

If K and X are expressed as functions of O

$$\frac{1}{K(1 - X)} = f_0(O) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

$$\frac{X}{1 - X} = f_1(O) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

and equation (18) can be re-written as follows.
In case we make the mathematical operation block diagram of equation (22) it will be as illustrated in Fig. 10.

$$O = f_0(O) \cdot s - f_1(O) \cdot I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

2. EARTHQUAKE RESPONSE SIMULATOR

Japan which has many earthquakes is destined to construct its building earthquake proof. For a long period of time earthquake proof buildings were designed to withstand the earthquake force evaluated statistically and the skelton and materials were decided accordingly. However, designs for super high structure buildings of 20 to 30 stories and extremely important buildings such as atomic reactors were based on the fact that earthquakes show a dynamic response which is the proper approach, and recently, dynamic analysis or response analysis are made as a rule.

As for the content of the analysis response, for irregular waves such as earthquake vibrations, the displacement and stress of the buildings which are replaced by multi-mass points are calculated from elasticity to plasticity. Thus, a considerably high calculating technique will be required. The reason that calculation of high technique has become possible is due to the recent developments in electronic computers. Such calculations are almost all done by digital computers in the United States, but in Japan, the analog type and digital type are used together.
Especially, in the initial stage a few years ago, development and research work was done by the analog computers.

As the first application of simulation in this field, there is an analog computer named SERAC specializing in the response analysis of earthquakes.
SERAC is a slow speed type electronic tube analog computer, and it is the abbreviation of STRONG EARTHQUAKE RESPONSE ANALOG COMPUTER.
In 1961, the Toyo Rayon Technological Research Assistance, "Strong Earthquake Response Analysis Committee" (Representative: Dr. Kiyoshi Muto, Tokyo University) planned this, and the equipment was manufactured by the Hitachi, Ltd.

The vibration and damage conditions of buildings at the time of great earthquakes are analyzed dynamically, and from the stand point of dynamics, it was utilized with the aim of studying the earthquake proof designing methods for super high structure and modern constructions.
At present, in order to make complex calculations of structures having extremely large amounts of mass points accurately, the digital computers are used. However, in case of

studies for obtaining the tendency of the characteristics, or in the stage of trial designs, since trial calculations are made a large number of times by changing the parameters, the analog computers are put to use from the standpoint of convenience and speed, even at the present time.
Furthermore, it is estimated that henceforth, for the calculation of the constants and specific values of the vibrational system model, the digital computers will be utilized, and for the parametric studies, analog computers will be utilized. For instance, if the building cycle is changed and the acceleration added to the building is investigated (In other words, the ration between the force added to the building and the weight of the building), as the building cycle becomes longer, it was found that the acceleration becomes very small.
As a result, a good example of analog computer utilization may be said to be the search for possibilities of earthquake proof designs for super high structures. Furthermore, in order to obtain the stress at each floor and to obtain the vibration quantitatively and accurately, the digital computer is more suitable.

2.1. Simulation based on Analog Computers (Example of SERAC)

As almost all vibrational phenomena can be expressed by differential equations, it is well known that not only the vibration of buildings caused by earthquakes but also other vibrational phenomena are quite suitable for the analysis by analog computers.
Especially, in the case where a phenomenon is precisely analysed. Since there are non-linear factors such as backlash and plastic deformation, it is more practical to utilize computers.
When we consider the simplicity of the program, the speed in obtaining solution and the economics, it is not an exaggeration to say that it is one of the most suitable fields of application for analog computers.
As the outstanding characteristics of SERAC, the following points can be given.

(1) A random wave like earthquake wave can be bed in as inputs. (In order that recorded wave shapes of great earthquakes can be used directly.)

(2) Response calculations of vibrational systems which have bi-linear type non linear type non linear characteristics, and which accompany hysteresis can be made. (In order to simulate a condition in which the building undergoes a great deformation, and part of it is damaged and has entered the range of plastic deformation.)
Next, an explanation will be made in the following paragraphs on the method in which earthquake response analysis is done by the SERAC.

2.1.1. Linear Response of One Mass Point System

The equation of motion in which one mass point system having a viscosity damping property like the one shown in Fig. 11, receives an earthquake vibration is as follows:

$$m\frac{d^2y}{dt^2} + c\left(\frac{dy_0}{dt} - \frac{dy}{dt}\right) + K(y - y_0) = 0 \quad \text{......................} \quad (23)$$

where

m : Mass
t : Time
y : Absolute displacement of Mass Point
y_0 : Displacement of Fixed Point i.e. earthquake
c : Viscosity Damping Coefficient
K : Spring Constant

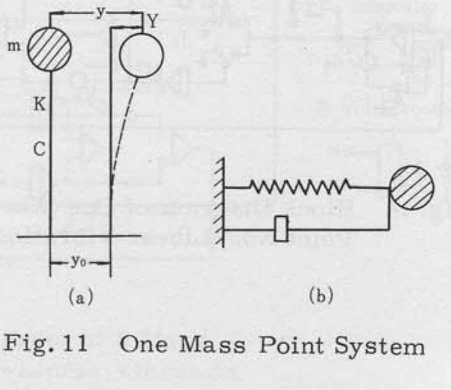

Fig. 11 One Mass Point System

In case this relation is re-written into an equation showing the relative displacement Y against the fixed point of the mass point, it will be as follows:

$$m\ddot{Y} + c\dot{Y} + KY = -m\ddot{y}_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

c expresses the differential calculation of time t.

In order to solve this by the analog computer, this is changed to

$$\ddot{Y} = -\ddot{y}o = \frac{c}{m}\dot{Y} - \frac{K}{m}Y$$

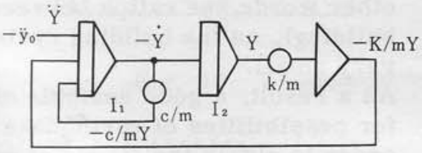

Fig. 12 Block Diagram for One Mass Point System

and the block diagram shown in Fig. 12 is used. yo is the record of the earthquake acceleration. Records which were taken by a strong earthquake meter designed especially for great earthquakes was used. This was enlarged and put through the equipment via the photographic curve reader (Photo-electric type function generator)
As for the time scale factor, 5, 10, or 20 is used. Since the continuing time of the earthquake is from 10 seconds to 20 seconds, the time required for the mathematical operation is about 3 to 4 minutes at the most.

2.1.2. Non-Linear Response of One Mass Point System

The non-linear characteristics handled by SERAC are chiefly of a bi-linear type having hysteresis as mentioned at the beginning, and are as shown in Fig. 13. Besides these characteristics, there is also a tri-linear type, but the block diagram would be extremely complicated.
The equation of vibration can be expressed as follows.

$$mY + CY = Q(Y) = -my_0 \quad \ldots \quad (25)$$

The term Q(Y) on the above equation is the so-called restoration force, and it is a function of the relative displacement Y. (In Fig. 13, this is expressed by δ)
The mathematical operations performed by SERAC are done in accordance with the block diagram shown in Fig. 14.

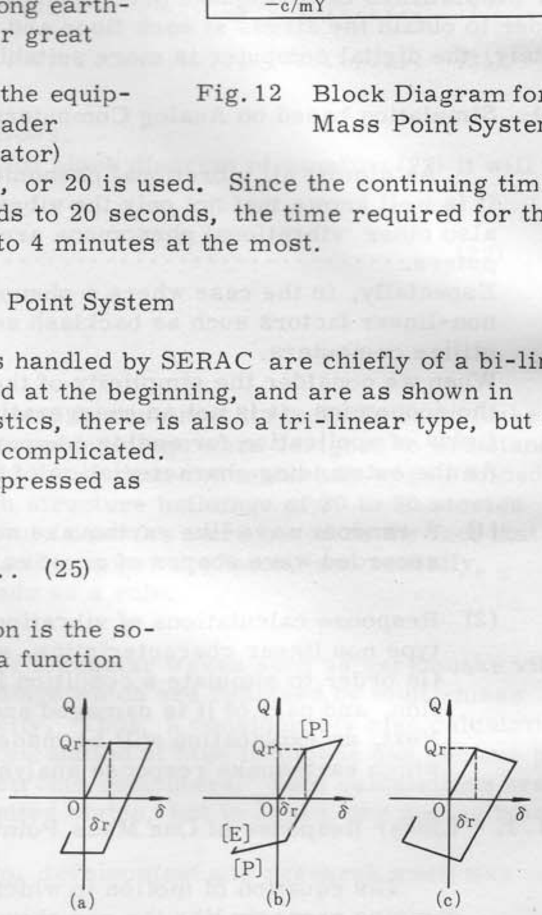

Fig. 13 Non-linear characteristics

$\delta \equiv Y \equiv y - y_0$

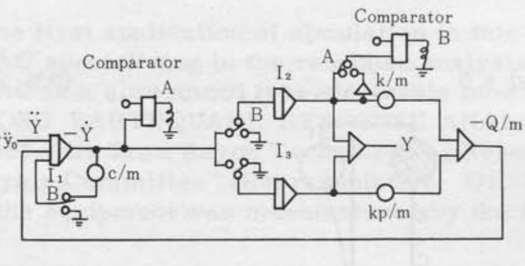

Fig. 14 Block Diagram of One Mass Point Non-Linear Vibration

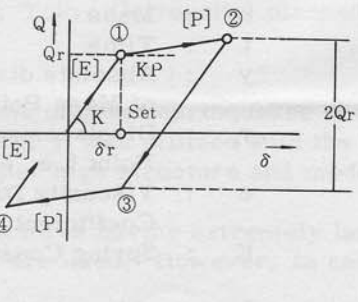

Fig. 15 Division of Elasticity and Plasticity in the Restoring Force

If we explain in detail the example shown in Fig. 13 (b), as shown in Fig. 15, this is divided into the elastic portion E and the plastic portion P.
The E portion is represented by the loop shown in the top portion of Fig. 15, and the P portion is calculated from the loop shown in the lower portion of the same diagram.
For the changeover of the two, mechanical relays are used. If we start from 0 point in Fig. 15, at first we will pass through the E loop (the same as the elasticity block diagram shown in Fig. 12).
However, as the deformation increases and point (1) is reached, (E) will be cut off and loop (P) is obtained.
If Q becomes a voltage equivalent to Q by the comparator (B), relay B will be made to actuate. After advancing (P) for a while, E will appear again when the relative speed is 0. (Point (2) in Fig. 15)
This is done by actuating the relay by comparator (A).
Henceforth, the same procedure is repeated.
The simulation of elasticity and plasticity by the mechanical relay system operate quite accurately at a time lag which can be neglected. Thus, it is utilized with great success.

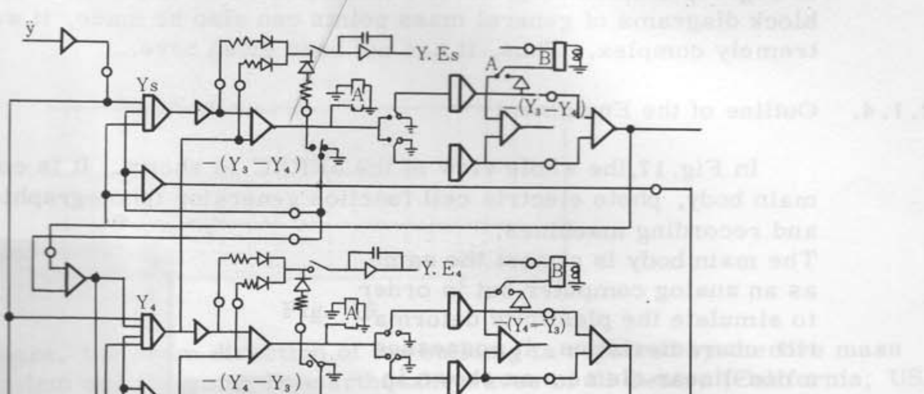

Fig. 16 Block Diagram of 5 Mass Point Non-Linear Vibration

2.1.3. Response of Multi Mass Point

In case of a shearing force type multi-mass point, the number of circuits shall be made to be the same as the number of mass points, and in the same way as the one mass point system, they shall be combined.
For reference, if an equation for linear vibration of the i-th mass point is made, it will be as follows:

$$
\begin{aligned}
& m_i \ddot{Y}_i + c_i(\dot{Y}_i - \dot{Y}_{i-1}) \\
& \quad + c_{i+1}(\dot{Y}_i - \dot{Y}_{i+1}) \\
& \quad + K_i(Y_i - Y_{i+1}) \\
& \quad + K_{i+1}(Y_i - Y_{i+1}) \\
& = - m_{yo} \cdots\cdots\cdots \quad (26)
\end{aligned}
$$

In Fig. 16, a block diagram of 5 mass point non-linear case is shown. Although block diagrams of general mass points can also be made, it would become extremely complex. Thus, it has not been given here.

2.1.4. Outline of the Equipment

In Fig. 17, the whole view of the SERAC is shown. It is composed of the main body, photo electric cell function generator (photographic curve reader), and recording machines.
The main body is almost the same as an analog computer but in order to simulate the plasticity deformation characteristics, it possesses a non-linear element as shown in Fig. 13 and Fig. 14.
The composing elements of the main body are as shown in Table 1.
On the pre-patch board of the main body, the above mentioned simulation circuit is patched, and simulation is made.

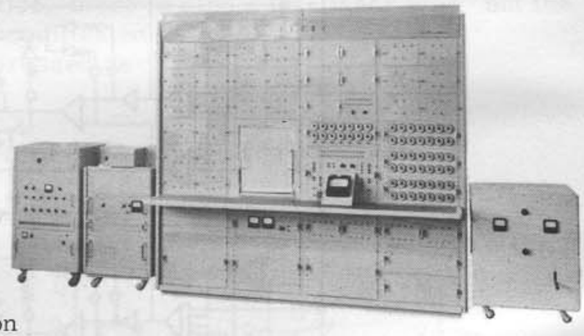

Fig. 17

The photo-electric cell type function generator is used for feeding earthquake waves into the simulation circuit. The earthquake waves made on the recording paper are made to undergo pulse width modulation by tracing with a light spot, and by restoring the output, it is then taken out as an analog voltage reading.

Table1. Composition of Elements

Elements	Quantity
Integrator	17
Special Integrator	5
Adder	26
Inverter	16
Potentiometer	56
Voltage Comparator	12

Fig. 18 Photoelectric Curve Tracer

The external view is shown in Fig. 18.
As for the recorded earthquake waves, strong earthquake records of the U.S. which was prepared by Prof. Borg, and Japanese records prepared by the

Technical Research Division of Kashima Kensetsu Ltd. are used.
The devices for recording the solutions (simulation results for the displacement and stress of structures) are done by the ordinary pen oscillographs.
Fig. 19 shows an example of a solution.

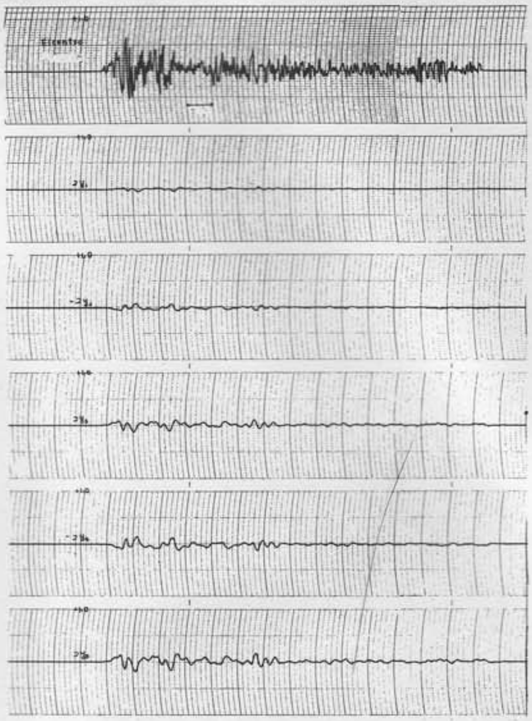

Fig. 19

In this case, the beam direction of the building is replaced by the five mass point system and the recorded earthquake waves of El Centro (California, USA, 1940) were put in and simulated. Furthermore a report of this was made.
The waves were as shown below.

No. 1 Channel	Earthquake wave shape	y_0
No. 2 Channel	First Floor Relative Displacement	Y_1
No. 3 Channel	Second Floor Relative Displacement	$Y_2 - Y_1$
No. 4 Channel	Third Floor Relative Displacement	$Y_3 - Y_2$
No. 5 Channel	Fourth Floor Relative Displacement	$Y_4 - Y_5$
No. 6 Channel	Fifth Floor Relative Displacement	$Y_5 - Y_4$

2. 2. Simulation based on Digital Computer

The only difference is the computer being used. As for the equations which express the vibration of the structure, the same wave used for the analog computer simulation were calculated by the digital computer. In case the analog computer is used a simulation in which the vibration is about ten times as slow as the actual earthquake is obtained.

Contrary to this, in case the digital computers are used, several hours of calculation will be required. However, in case of analog computers, in order to make analysis of super high structures, if mass points are substituted for the number of floors, a large scale computer will have to be set in proportion to the number of pass points. On the other hand, in case of the digital computer, a similar calculation can be made so long as the time of calculation is changed.
Therefore, if precise data is required such as in the case of making confirmation of the earthquake resistance obtained by calculations, simulations based on digital computers are more suitable.